\[ L(\vec{r}_q - \vec{r}_p, \vec{v}_q - \vec{v}_p) = \]
\[ = \sum \frac{m_i \vec{r}_i^2}{2} - \sum \frac{U(\vec{r}_i - \vec{r}_j)}{\text{lag}} \]

**Gallilean:**

\[ \vec{r}_i \rightarrow \vec{r}_i + \vec{V}_0 t \]
\[ \vec{v}_i \rightarrow \vec{v}_i + \vec{V}_0 \]

Symmetry \(\equiv\) active, two variant \(\not\equiv\) not necessarily \(L\) !

\[ L(\{\vec{r} + \vec{V}_0 t, \vec{r} + \vec{V}_0 t\}) - L(\{\vec{r}, \vec{r}\}) = \]
\[ = \sum m_i \cdot \dfrac{d}{dt} (\vec{r}_i \cdot \vec{V}_0) \]

L.h.s:

\[ = \sum \dfrac{d}{dt} \left( \sum m_i \vec{r}_i \cdot \vec{V}_0 \right) \]

R.h.s., for any \(\vec{V}_0\),

including small

\[ = \dfrac{d}{dt} \left( \vec{P}_{cm} \cdot \vec{V}_0 \right) \]

\[ = \dfrac{d}{dt} \left( \vec{P} \cdot \vec{V}_0 t \right) \]
Gallilean invariance $\Rightarrow$

\[ \frac{d}{dt} \left( \frac{\vec{P}}{\mu} \cdot \vec{v}_0 - \vec{P}_t - \vec{v}_0 \right) = 0 \]

\[ \Rightarrow \frac{\vec{P}}{\mu} \cdot \vec{v}_0 - \vec{P}_t - \vec{v}_0 = \frac{\vec{R}_0}{\mu} \cdot \vec{v}_0 \]

constant vector $\times \vec{v}_0$

\[-\vec{R}_0 \Rightarrow \vec{R}_{cm} = \frac{\vec{P}_t + \vec{R}_0}{M} \]

conserved quantity due to Gallilean invariance.

Final: Noether theorem $\leftrightarrow$ active invariance

for Galilean $\Rightarrow$ CM moves in straight line
Claim: d d.o.f.

\( \Rightarrow \) at most 2d - 1 independent \( S \) of motion.

2d = # initial conditions

\( d \) q's + d \( \dot{q} \)'s (and cons.)

- but trajectory depends to as well as one can make
to form one of arbitrary constants.

\[ r^2 = r_0^2 + v_0^2 (t - t_0) \]

\[ = (r_0 - v_0 t_0) + v_0 t. \]

\( \tau \) shift to \( \equiv \) shift of \( r_0 \) to origin of \( v_0 \).

So at most 2d - 1 indep. \( S \) of motion.

\( \text{Hence:} \quad E, \quad \vec{p}, \quad \vec{r} \quad \text{with} \quad \begin{cases} E^2 = \vec{p}^2 / 2m, \\ \vec{p} \cdot \vec{r} = 0 \end{cases} \)

\( \vec{p} \to \vec{v}_0 \)

\( \vec{r} \to \text{only } \vec{r}_0 \perp t \to \vec{v}_0 \)

\( \text{If } "d" \text{ } \text{indifferent } \)

\( \text{"} > d \text{" } \text{super-indifferent } \)

\( \text{"} 2d - 1 \text{" } \text{max. symmetric } \)
Comments on uses of symmetry...

Generally: symmetries \implies conservation laws

\downarrow

use to simplify

solving the problem

(since these quantities are guaranteed not to change-

- whatever they are @

beginning of motion \( t=t_0 \)

they retain this value)

\underline{Ex's.}

\( 0 \) - free particle

\[ L = \frac{\vec{p} \cdot \vec{v}^2}{2} , \quad E, \vec{p}, \vec{M} \]

all conserved, \( 7 \) quantities

however not all independent

\[ E = \frac{\vec{p}^2}{2M} \quad (= \frac{\vec{m} \cdot \vec{v}^2}{2}) \]

\[ \vec{p} \cdot \vec{H} = 0 \quad \text{(since } \vec{H} = \vec{r} \times \vec{p} \text{)} \]

so only 5 are independent: as it should be -

- since there are 6 initial conditions \( (\vec{r}_0, \vec{v}_0 = \vec{r}_0^2) \)

that determine solution \( \vec{r} = \vec{r}_0 + \vec{v}_0 (t - t_0) \)
But the value of to is arbitrary.

It can not be determined by knowing \((E, \vec{r}, \vec{p})\).

On the other hand, shift of \(t\) \(\Leftrightarrow\) shift of \(r_0\)

(in dir'n \(\vec{v}_0\))

Hence at most can have 5 independent
integrals of motion (and that's true!)

Generally, if "d" degrees of freedom
have "2d" initial conditions (initial \(q_i's\) \& \(\dot{q}_i's\))

but one always corresponds to a choice of \(t\).

\[ \rightarrow \text{So at most } 2d - 1 \text{ independent} \]
integrals of motion for "d" d.o.f.

Def. If more than "d" independent
integrals of motion "superintegrable" (if "d" "integrable"

If "2d - 1": "maximally superintegrable"

Ex. 2: 3 particles in central \(U(1, \vec{r}_1 - \vec{r}_2, \vec{r}_3 - \vec{r}_2)\)

potential: \((E, \vec{p}, \vec{r}_i)\), again

3 integrals of motion
Combining ideas of CM frame + angular momentum conservation \(\Rightarrow\) useful to simplify motion in central potentials, say (Kepler problem, H = above)

\[\text{as you can tell we're in Ch. III of L&L, §12 (skipped thru "mechanical similarity" and "one-dim motion" -- latter is way too simple, former will mention later!)}\]

In an arbitrary frame two particles \(w_1, w_2, \vec{r}_1, \vec{r}_2\) interacting via \(U(\vec{r}_1 - \vec{r}_2)\) have

\[L = \frac{1}{2} w_1 \vec{v}_1^2 + \frac{1}{2} w_2 \vec{v}_2^2 - U(\vec{r}_1 - \vec{r}_2), \quad (\vec{v}_i = \frac{\vec{p}_i}{m_i})\]

as we know from discussion of \(R_{CM}\)

\[R_{CM} = \frac{w_1 \vec{r}_1 + w_2 \vec{r}_2}{w_1 + w_2}\]

let's take \(R_{CM}\) as the origin of coordinates, i.e. \(w_1 \vec{r}_1 + w_2 \vec{r}_2 = 0\) in this frame

and let: \(\vec{r} = \vec{r}_1 - \vec{r}_2\), then
\[ \vec{r}_1 = \vec{r} + \vec{r}_2 \text{ plug into } w_1 \vec{r}_1 + w_2 \vec{r}_2 = 0 \implies \]
\[ w_1 \vec{r}_1 + w_1 \vec{r}_2 + w_2 \vec{r}_2 = 0 \implies \vec{r}_2 = -\frac{w_2}{w_1 + w_2} \vec{r} \]
\[ \text{since } \vec{r}_1 \cdot \vec{r} + \vec{r}_2 + \vec{r}_1 = \vec{r} - \frac{w_1}{w_1 + w_2} \vec{r} = \frac{w_1 + w_2 - w_1}{w_1 + w_2} \vec{r} = \frac{w_2}{w_1 + w_2} \vec{r} \]

So in c.m. we express both positions in relative position.

\[ \begin{array}{c}
\text{Qu: as pictured, which mass is larger, } w_1 \text{ or } w_2 \text{?} \\
\text{origin = C.M.}
\end{array} \]

plug \( \vec{r}_1 \) & \( \vec{r}_2 \) expressed via \( \vec{r} \) into \( L \) of p.(58):

\[ L = \frac{1}{2} w_1 \left( \frac{w_2}{w_1 + w_2} \vec{r} \right)^2 + \frac{1}{2} w_2 \left( -\frac{w_1}{w_1 + w_2} \right)^2 \vec{r}^2 - U(l_1^2) \]

\[ = \frac{1}{2} (\vec{r})^2 \frac{w_1 w_2^2 + w_2 w_1^2}{(w_1 + w_2)^2} - U(l_1^2) = \frac{1}{2} (\vec{r})^2 \frac{w_1 w_2}{w_1 + w_2} - U(l_1^2) \]
So in c.w. $L$ looks simple -

- like that of a particle of mass

$$W_{\text{reduced}} = \frac{w_1 w_2}{w_1 + w_2} \quad (\rightarrow w_2 \text{ if } w_2 \gg w_1)$$

in a potential ("external") $U(r)$:

$$L = \frac{\hbar^2}{2} \cdot \frac{r^2}{r^2} - U(r)$$

(for the moment I'll drop subscript "reduced")

(for brevity)

(Note: so for $U(r)$ vs $U(r)$ has not been important.)

So, if we solve for $\vec{r}(t)$, can find $r_1(t)$ & $r_2(t)$ by using the relations of p.59.

If we have a "central" field

$$U(r) = \frac{\text{const}}{r} \quad \text{(Coulomb, Newton...)}$$

of

$$U(r) = \frac{\text{const}}{r} \quad \text{(Yukawa - pions - N-P)}$$
\( U(r) \) 

short-range attraction

- long-range attraction

- repulsion

Typical shape of interatomic interaction.

things simplify a bit (at least, a bit)

thing is \( L \) is invariant under rotation (in C.M., that is) \( \Rightarrow \) hence \( \hat{M} \) is conserved.

\[ \hat{M} = \hat{p} \times \hat{p} \] = constant (means time-independent - value fixed by initial condition)

Hence, since \( \hat{p} \cdot (\hat{p} \times \hat{x}) = 0 \) (vector product of \( \hat{p} \) with anything is \( \perp \) to \( \hat{p} \))

and since \( \hat{M} = \text{constant vector} \)

we have that \( \hat{p} \cdot \hat{M} = 0 + \hat{M} = \text{constant} \)
but this means that motion lies in a plane: \[ \overrightarrow{M} \]

\[ \nabla \text{ trajectory is PLANAR} \]

So we choose coordinates such that \[ \overrightarrow{M} / \overrightarrow{2} \]

Then \( r^2 \) lying in a plane means \( \Theta = 0 \) \( (\Theta = \frac{\pi}{2}) \)

so we have \( \overrightarrow{r}^2 = \overrightarrow{r}^2 + \overrightarrow{r} \cdot \overrightarrow{p}^2 \)

\[ L = \frac{m}{2} \overrightarrow{r}^2 + \frac{mr^2}{2} \overrightarrow{p}^2 - U(r) \]

This is simply related to \( M_2 \):

Recall that \( M_2 = x \overrightarrow{y} m - y \overrightarrow{x} m \) \( (x(v_y m) - y(v_x m)) \)

\[ = (r \cos \phi) m(r \sin \phi)^2 - (r \sin \phi)(r \cos \phi)^2 m = \]

\[ = m r \cos \phi r \sin \phi + m r \cos \phi r \cos \phi \overrightarrow{p} \]

\[ - r \sin \phi r \cos \phi + m r \sin \phi r \sin \phi \overrightarrow{p} \]

\[ = m r^2 \overrightarrow{p} (\cos \phi + \sin \phi) = m r^2 \overrightarrow{p} \]
\[
\begin{align*}
\text{hence} & \quad mr^2 \dot{\phi} = M_2 = \text{const.} \implies \dot{\phi} = \frac{M_2}{mr^2} \\
\text{but L has} & \quad mr^2 \dot{\phi}^2 = mr^2 \frac{M_2^2}{mr^4} = \frac{M_2^2}{m r^2}\\
\text{hence, writing L into } M_2 \quad \text{(conserved quantity)}
\end{align*}
\]

we have

\[
L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} \frac{M_2^2}{r} = U(r)
\]

\[
\uparrow
\]

a 3d problem \iff a 4d problem

What else is conserved? \dash\dash\dash well, energy, of course. \rightarrow so let's use that \rightarrow

we know

\[
E = \frac{1}{2} (m \dot{r}^2 + m r^2 \dot{\phi}^2) + U(r) = \frac{1}{2} m \dot{r}^2 + \frac{m r^2 \dot{\phi}^2}{2} + U(r)
\]

\[
\implies \frac{M_2^2}{2 m r^2} \quad \text{as we did above}
\]

\[
\text{so} \quad E = \frac{1}{2} m \dot{r}^2 + \frac{M_2^2}{2 m r^2} + U(r)
\]

\[
\text{(Drop } r \text{ from } M_2 \implies )
\]
For this problem, we don't even need \( E-\mathcal{L} \), equation => this is because we have 2 variables that describe the trajectory \((x(t), y(t))\) a line in the plane \( \mathbb{R}^2 \) 
but we also have 2 integrals of motion \((E, H)\) => so we can solve it to: \( E \neq M \)!

However, we will \( \frac{dy}{dt} = \frac{M}{m \sqrt{r^2}} \) (typ of \( \mathcal{L} \))

Since \( E = \frac{m}{2} \dot{r}^2 + \frac{M^2}{m^2} + U(r) \)

Hence \( \dot{r}^2 = \frac{2E - 2U(r) - \frac{M^2}{m^2}}{r^2} \)

Of course, this is always \( > 0 \) for classically allowed motion this is always \( > 0 \)

\[*\] \( \dot{r} = \frac{dr}{dt} = \sqrt{\frac{2}{m} (E-U(r)) - \frac{M^2}{m^2 r^2}} \)

Hence \( dt = \frac{dr}{\sqrt{\frac{2}{m} (E-U(r)) - \frac{M^2}{m^2 r^2}}} \)
Solving (*), give \( r(t) \) for given \( E \) & \( M \) (sec)

particle comes from \( \infty \), reaches \( r = 0 \), then sign of \( r \) changes \((t \to +)\) if particle leaves off \( r = 0 \)

But it is even more advantageous to find

equation for path - simply replace \( dt \) by \( dy \)

\[
\Rightarrow \quad \frac{u^2}{r^2} \, dy = \frac{dr}{M \left( \frac{2}{m} (E-U(r)) - \frac{M^2}{u^2 r^2} \right)^{1/2}}
\]

Here \( dy = \frac{M}{u^2} \frac{dr}{r^2 \left( \frac{2}{m} (E-U(r)) - \frac{M^2}{u^2 r^2} \right)^{1/2}} \)

\[
dy = \frac{M}{r^2} \frac{dr}{\sqrt{2m(E-U(r)) - M^2/r^2}}
\]

so \( y = \int \frac{M}{r^2} \frac{dr}{\sqrt{2m(E-U(r)) - M^2/r^2}} + \text{const.} \)

Equation for trajectory as \( y = y(r) \)

I will study integration in some important examples later!
\[ L = \frac{1}{2} \mu \mathbf{P}_{\text{CM}}^2 + \frac{\mu}{2} \mathbf{\dot{\mathbf{\gamma}}}_1^2 - U(\mathbf{\gamma}_1^2) \]

\[ \mu = \mu_1 + \mu_2 \]

Total mass

\[ \mu = \frac{\mu_1 \mu_2}{\mu_1 + \mu_2} \]

"Reduced mass"

Relative motion

\[ \mathbf{P} = \mu \mathbf{P}_{\text{CM}} \]

\[ \mathbf{\dot{\mathbf{\gamma}}} = \mathbf{\dot{\mathbf{\gamma}}}_1 \times \mathbf{\dot{\mathbf{\gamma}}}_2 \]

wrt \( \mathbf{P} = 0 \)

\[ E = E_{\text{CM}} + E_{\text{internal}} \]

\[ E_{\text{CM}} = \frac{\mu}{2} \mathbf{P}_{\text{CM}}^2 + \frac{\mu}{2} \mathbf{\dot{\mathbf{\gamma}}}_1^2 + U(\mathbf{\gamma}_1^2) \]

separately conserved, as \( E_{\text{CM}} = \frac{\mathbf{P}_{\text{CM}}^2}{2\mu} \)

Since \( \mathbf{\dot{\mathbf{\gamma}}} = \text{const} \Rightarrow \mathbf{\dot{\mathbf{\gamma}}} \times \mathbf{\dot{\mathbf{\gamma}}} = \text{const} \) is conserved on its own

both \( \mathbf{\dot{\mathbf{\gamma}}} \) and \( \mathbf{\dot{\mathbf{\gamma}}} \) are \( \perp \) to \( \mathbf{\dot{\mathbf{\gamma}}} \), so motion is PLANAR.

(Plane is determined by \( \mathbf{\dot{\mathbf{\gamma}}} \times \mathbf{\dot{\mathbf{\gamma}}} \))
Since now we can take
\[
\vec{r} = (r, \theta, \phi) \text{ in plane } 1 = \hat{\mathbf{r}}
\]

& both \( E_{\text{kinetic}} = \frac{1}{2} \hat{M} \cdot \frac{\ddot{r}^2}{r^2} + \frac{1}{2} \hat{M} \cdot r^2 \dot{\theta}^2 + U(r) \)

& \( |\vec{r}| = \hat{M} \cdot r^2 \dot{\phi} \) are conserved

\[ \] to find \( r(t) \) & \( \phi(t) \)

there's no need to solve any diff eqns.!!

\[ \phi = \frac{|\vec{r}|}{\hat{M} \cdot r^2}, \ E_{\text{tot}} = \frac{1}{2} \dot{r}^2 + \frac{|\vec{r}|^2}{2 \hat{M} \cdot r^2} + U(r) \]

\[ \] like 1 dim motion .

(small "centrifugal barrier")

(potential)

recall
\[
\left( \frac{d}{dt} \right)^2 = \frac{2}{\hat{M}} \left( E_{\text{tot}} - \frac{|\vec{r}|^2}{2 \hat{M} \cdot r^2} - U(r) \right)
\]

solution in quadrature

\[
dt = \sqrt{\frac{2}{\hat{M}}} \frac{dr}{\sqrt{E_{\text{tot}} - \frac{|\vec{r}|^2}{2 \hat{M} \cdot r^2}} - U(r)}
\]

(use \( dt = \frac{\sqrt{\hat{M}} \cdot dr}{|\vec{r}|} \) to find \( \phi = \frac{|\vec{r}|}{\hat{M} \cdot r^2 \sqrt{2}} \cdot \int dr \rightarrow \text{pol} \))
Exercise (c.5) - gets even better if \( U = \frac{x}{r} \)

(Keppler or H. atom)

Tums out there's more symmetry!!

\( E, \vec{p}, \vec{M}, \vec{A} \)

\( \rightarrow \) Raugey tensor vector

(Which allows us to skip doing \( dt = \sqrt{\frac{2}{V E - E^2 - U(r)}} \))

Can get orbits w/ it!

This! <

\[ \vec{A} = \vec{p} \times \vec{M} - \alpha \vec{F}, \quad (\text{all refer to CM frame} \, \, \vec{F}, \vec{p}) \]

\( \vec{A}^2, \vec{F} = 0, \quad \vec{A}^2 = \text{(function of } E \, \text{& } |\vec{M}|^2) \)

so really one more independent \( f \) of motion - but really all one needs to solve for trajectory of relative coordinate. (The relative coordinate problem is "maximally superintegrable": 3 dof, 5 dof motion \( (E, M, A^2) \) \[ \frac{4}{4} + \frac{1}{1} = 5 \)
Kepler orbits via "Runge-Lenz" vector

\[ \vec{J} = \vec{r} \times \vec{v} \quad m, \quad \vec{J} = \text{const.} \quad \forall \ U(r) \]

(1) \[ \frac{d}{dt} \left( \vec{v} \times \vec{J} \right) = \frac{d\vec{J}}{dt} = \frac{d}{dt} \left( \vec{r} \times \vec{v} \right) \]

(2) \[ m \frac{d\vec{v}}{dt} = -\vec{\nabla} U(r) = -\frac{dU(r)}{dr} \vec{r} \]

(3) \[ \vec{\nabla} r = \frac{\vec{r}}{r} \quad \left( \vec{\nabla} \sqrt{x^2 + y^2 + z^2} = \left( \frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right) \right) \]

So in any radial \( U(r) \) we have (2) & (3) imply (1):

\[ \Rightarrow \frac{d}{dt} \left( \vec{v} \times \vec{J} \right) = -\frac{1}{m} \frac{dU(r)}{dr} \vec{r} \times \left( \vec{r} \times \vec{v} \right) m \]

\[ = -\frac{U'(r)}{r} \frac{\vec{r}}{r} \times \left( \vec{r} \times \vec{v} \right) = - \frac{U'(r)}{r} \vec{v} \]

For Coulomb/Kepler

\[ U(r) = \frac{1}{r} \quad U'(r) = -\frac{1}{r^2} \quad \text{use } A \times (B \times C) = 
\]

\[ = B(A \cdot C) - C(A \cdot B) \]

\[ \vec{v} = -\frac{U''(r)}{r} \left[ \vec{r} \left( \vec{r} \cdot \vec{v} \right) - \vec{v} \vec{r}^2 \right] = \Delta \left( \frac{\vec{r} \left( \vec{r} \cdot \vec{v} \right)}{r^3} - \frac{\vec{v}}{r} \right) \]

\[ \frac{\vec{r}^2}{r} = \frac{\vec{v}^2}{r}, \quad \frac{d}{dt} \left( \frac{\vec{r}^2}{r} \right) = \frac{\dot{r} \vec{r} - \vec{r} \dot{r}^2}{r^2} = \frac{\dot{v}}{r} - \frac{\left( \vec{r} \cdot \vec{v} \right) \vec{r}^2}{r^3} \]

\[ \vec{v} = \frac{d}{dt} \vec{r}/r = \frac{\vec{r} \cdot \vec{v}}{r^2} = \frac{\vec{r} \cdot \vec{v}}{r} \]
So we have

$$\frac{d}{dt} (\vec{v} \times \vec{M}) = \alpha \frac{d}{dt} \vec{r}$$

or

$$\frac{d}{dt} (\vec{v} \times \vec{M} - \alpha \vec{r} \times \vec{r}) = \boxed{0} \quad \text{correct!}$$

(\alpha = G \frac{M_0 m}{r^2} \text{ for Kepler})

So \(\vec{v} \times \vec{M} = \alpha (\vec{r} \times \vec{e})\) is the general solution of (2)

A constant vector \(\alpha \vec{r} \times \vec{e}\)

multiply by \(\vec{M}\):

$$\Rightarrow \vec{M} (\vec{v} \times \vec{M}) = \alpha (\vec{r} \times \vec{e}) \times \vec{r} + \alpha \vec{M} \vec{e}$$

but $a = 0$

$$\Rightarrow \vec{e} \perp \vec{M}$$

lies in plane

multiply by \(\vec{r}\):

$$\vec{r} \cdot (\vec{v} \times \vec{M}) = \alpha (\vec{r} \cdot \vec{r} + \vec{r} \cdot \vec{e})$$

\(A \cdot \vec{C} = B \cdot (\vec{C} \times \vec{A}) = C \cdot (\vec{A} \times \vec{B})\)

$$\Rightarrow \vec{M} (\vec{r} \times \vec{v}) = \frac{\vec{M}^2}{m}$$

$$\frac{\vec{r} \cdot \vec{r}}{r} = r \quad \vec{e} \cdot \vec{r} = e r \cos (\phi - \phi_0)$$
\[ \vec{r}^2 = \mu a \left( r + e \cos(\varphi - \varphi_0) \right) \]

so

\[ r = \frac{\left( \frac{\vec{r}^2}{\mu a} \right)}{1 + e \cos(\varphi - \varphi_0)} \]

equation of orbit.

\[ \Rightarrow \text{get } r \Rightarrow p = \frac{\mu a}{M^2} = \text{"latus rectum"} \]

\[ e = \text{eccentricity} = \sqrt{1 + \frac{2EM}{\mu a^2}} \]

\[ \frac{p}{r} = 1 + e \cos \varphi \quad \text{let } \varphi_0 = 0. \]

extra "hidden" symmetry of Kepler problem:

\[ E(0) : \text{so}(3) \rightarrow \text{so}(4) \]

\[ E(0) : \text{so}(3) \rightarrow \text{so}(3,1) \]

\[ \frac{dr}{dt} = \frac{m^2}{\mu a} \left( -1 \right) \frac{1}{(1 + e \cos \varphi)^2} e(-) \sin \varphi \frac{dy}{dt} \]

\[ M = m \frac{r}{2} \frac{dy}{dt} = \frac{M}{m^2} \]

\[ s \frac{dr}{dt} = \frac{M^3}{m^2 a^2 (1 + e \cos \varphi)^2} \frac{r^2}{r^2 - \frac{M^2}{m^2} \frac{r^3}{M^2} - \frac{M}{m^2} \frac{r}{2} \frac{d}{dt} \Rightarrow \text{get} \]

in this derivation, to find \( e \)

- find \( \Delta E \) of a solution -

\( E(0) \)