ADVANCED UNDERGRADUATE LABORATORY

EXPERIMENT 17

MAGNETIZATION AND TRANSITION TEMPERATURES OF SUPERCONDUCTORS

Revised: June 1988
By: Derek Manchester
John Pitre
INTRODUCTION

This experiment looks at the magnetization of superconductors as functions of the magnetic field and of temperature. This enables critical fields ($H_c$, $H_{c1}$, $H_{c2}$) and the transition temperature in zero field ($T_c$) to be obtained and this then provides enough information for all the important physical quantities related to superconductivity in a metal to be determined.

The metals used in this experiment are lead, and examples of a Type I superconductor, and niobium, a Type II superconductor. These two metals have been chosen, primarily because they both have transition temperatures just a few degrees above the boiling point of liquid helium, 4.2 K. Thus, measurements made at 4.2 K and a few degrees above can be used to give data which can be extrapolated up in temperature a "degree or two" if need be, to give the zero magnetic field of superconducting transition, i.e. $T_c$. Using metals with $T_c$ above rather than below 4.2 K avoids the necessity of lowering the temperature of the superconductors by using cumbersome pumping equipment.

For an appreciation of the importance of magnetization measurements in understanding and characterizing superconductors see the reprint from Shoenberg given in appendix II for Type I superconductors and Type II superconductors see Kittel and also Saint-James et al.

EXPERIMENTAL

This experiment involves the use of liquid helium, so all the laboratory rules and precautions about handling this liquid should be observed.

The general layout of the coils for measuring the magnetization is shown in figure 1.
The measuring coils are surrounded by a glass tube exchange gas chamber connected through a 1/4 inch diameter tube to the probe top plate. A valve at the top plate (Hoke) permits the connection of the glass chamber to the general internal space of the helium dewar. When the helium dewar contains liquid helium, this valve may be used to connect the glass chamber to the vapor over the liquid helium and thus opening this valve provides a simple way to admit clean helium exchange gas to the glass chamber and thus a way of ensuring that the coils for magnetization measurements are at the temperature of the liquid helium bath which is 4.2 K. For magnetization measurements at temperatures
above 4.2 K the Hoke valve on the top plate is closed and the Veeco Valve which provides communication with a mechanical pump is opened. This valve must not be opened when liquid helium is in the dewar unless the appropriately connected mechanical pump is operating.

The pressure/vacuum in the pumping line is indicated with sufficient precision by the dial gauge. A reading of "30 vac" corresponds to a 30 inch height of a mercury column and is sufficient indication that the mechanical pump is operating satisfactorily for the purpose of the experiment.

Temperature measurements are made using a silicon diode thermometer set in the copper block on which coils are mounted. The thermometer operates by passing a constant current from a constant current source through the diode. The voltage required for this constant current is temperature dependent. A calibration table and curves of voltage versus temperature for the diode are given in appendix I. Also included in appendix I is the operating information for the constant current source.

Connecting the glass chamber to the mechanical pump will provide sufficient vacuum insulation around the copper mounting block containing the coils to allow the temperature of the measuring block to rise. Probably some power in the heater will have to be used to aid this temperature change. With a little care, the heater power can be adjusted to give a temperature which is stable enough to make a measurement of the magnetization (time involved is about 2 minutes)

Terminal connections on the probe to the heater, diode and coils are given in figure 2. The terminal connections to the various coils are routed to a box and the terminal connections on the box are given in figure 3. Before any measurements are attempted, the circuitry for obtaining a response for the magnetization should be set up. Use the connections to the measuring coil containing brass in an arrangement which enables first the Nb coil and then the Pb coil to be connected in opposition to the winding sense of the brass coil. This can be accomplished using female-female banana plug couplings with the special leads provided. All of these coils have very closely the same number of turns and therefore electrical resistances. Thus when they are connected in opposition, the induced e.m.f. produced by "ramping" the magnetic field through the coils will be due to the effect of the magnetization of Nb or Pb.
Ramping the magnetic field is achieved by using the ramp generator connected to the modulation input on the magnet power supply (it should be already connected). The output of the magnetization coils can be fed through a milli-microvoltmeter to the Y terminals of the X-Y plotter. Try very low gain settings to start with on both voltmeter and plotter in order to avoid overdriving the plotter. Do as much of the circuit testing as possible at room temperature before transferring any liquid helium.
The actual transfer of liquid helium for the first time or two, at least, must be done under the supervision of a demonstrator.

INTERPRETATION OF THE RESULTS

The magnetization results have to be integrated to give an $M(H)$ curve (make sure that you understand why). The variation of $M(H)$ with temperature can give a value of $T_c$ together with values of $H_{c1}(T)$, $H_{c2}(T)$ and possibly $H_{c1}(0)$ and $H_{c2}(0)$. Lead is a Type I superconductor and thus only $H_c(T)$, $H_c(0)$ and $T_c$ are obtained. With these experimental data available, determine all the important parameters for the Type II superconductors that you can e.g. $\kappa$, $\xi$, $\lambda$ etc. For a Type I superconductor, you do not have enough information to determine such parameters from experimental values.

In interpreting your experimental data for PB you are looking at the magnetization of a short cylinder. For this situation, consult the reprint by Shoenberg in appendix II and give an explanation and perspective of your result.

REFERENCES

2. D. Saint-James, G. Sarma and E.J. Thomas, Type II Superconductivity, 1969. (QC 612 S8S2)
3. D. Shoenber, Superconductivity, 1952. (QC 611 S52)
APPENDIX I

CALIBRATION CURVES AND DATA FOR TEMPERATURE SENSITIVE SILICON DIODE
Calibration Curve for Silicon Diode (S/N 17547) - 4.0 K → 300 K

Voltage VERSUS Temperature

0.00E+000  1.50E+002  3.00E+002
0.00E+000   1.23E+000   2.46E+000
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Voltage</th>
<th>Temperature</th>
<th>Voltage</th>
<th>Temperature</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>2.4649</td>
<td>25.0</td>
<td>1.1745</td>
<td>135.0</td>
<td>0.83666</td>
</tr>
<tr>
<td>4.2</td>
<td>2.4506</td>
<td>26.0</td>
<td>1.1473</td>
<td>140.0</td>
<td>0.82308</td>
</tr>
<tr>
<td>4.4</td>
<td>2.4361</td>
<td>27.0</td>
<td>1.1307</td>
<td>145.0</td>
<td>0.80945</td>
</tr>
<tr>
<td>4.6</td>
<td>2.4215</td>
<td>28.0</td>
<td>1.1203</td>
<td>150.0</td>
<td>0.79579</td>
</tr>
<tr>
<td>4.8</td>
<td>2.4068</td>
<td>29.0</td>
<td>1.1129</td>
<td>155.0</td>
<td>0.78209</td>
</tr>
<tr>
<td>5.0</td>
<td>2.3921</td>
<td>30.0</td>
<td>1.1072</td>
<td>160.0</td>
<td>0.76835</td>
</tr>
<tr>
<td>5.5</td>
<td>2.3552</td>
<td>32.0</td>
<td>1.0988</td>
<td>165.0</td>
<td>0.75455</td>
</tr>
<tr>
<td>6.0</td>
<td>2.3191</td>
<td>34.0</td>
<td>1.0924</td>
<td>170.0</td>
<td>0.74069</td>
</tr>
<tr>
<td>6.5</td>
<td>2.2842</td>
<td>36.0</td>
<td>1.0868</td>
<td>175.0</td>
<td>0.72676</td>
</tr>
<tr>
<td>7.0</td>
<td>2.2504</td>
<td>38.0</td>
<td>1.0817</td>
<td>180.0</td>
<td>0.71286</td>
</tr>
<tr>
<td>7.5</td>
<td>2.2177</td>
<td>40.0</td>
<td>1.0769</td>
<td>185.0</td>
<td>0.69894</td>
</tr>
<tr>
<td>8.0</td>
<td>2.1860</td>
<td>45.0</td>
<td>1.0655</td>
<td>190.0</td>
<td>0.68495</td>
</tr>
<tr>
<td>8.5</td>
<td>2.1550</td>
<td>50.0</td>
<td>1.0542</td>
<td>195.0</td>
<td>0.67088</td>
</tr>
<tr>
<td>9.0</td>
<td>2.1252</td>
<td>55.0</td>
<td>1.0428</td>
<td>200.0</td>
<td>0.65680</td>
</tr>
<tr>
<td>9.5</td>
<td>2.0967</td>
<td>60.0</td>
<td>1.0312</td>
<td>205.0</td>
<td>0.64275</td>
</tr>
<tr>
<td>10.0</td>
<td>2.0694</td>
<td>65.0</td>
<td>1.0193</td>
<td>210.0</td>
<td>0.62871</td>
</tr>
<tr>
<td>11.0</td>
<td>2.0163</td>
<td>70.0</td>
<td>1.0071</td>
<td>215.0</td>
<td>0.61468</td>
</tr>
<tr>
<td>12.0</td>
<td>1.9651</td>
<td>75.0</td>
<td>0.9975</td>
<td>220.0</td>
<td>0.60070</td>
</tr>
<tr>
<td>13.0</td>
<td>1.9116</td>
<td>77.35</td>
<td>0.9882</td>
<td>225.0</td>
<td>0.58677</td>
</tr>
<tr>
<td>14.0</td>
<td>1.8539</td>
<td>80.0</td>
<td>0.98205</td>
<td>230.0</td>
<td>0.57292</td>
</tr>
<tr>
<td>15.0</td>
<td>1.8114</td>
<td>85.0</td>
<td>0.96925</td>
<td>235.0</td>
<td>0.55917</td>
</tr>
<tr>
<td>16.0</td>
<td>1.7468</td>
<td>90.0</td>
<td>0.95631</td>
<td>240.0</td>
<td>0.54562</td>
</tr>
<tr>
<td>17.0</td>
<td>1.6776</td>
<td>95.0</td>
<td>0.94327</td>
<td>245.0</td>
<td>0.53224</td>
</tr>
<tr>
<td>18.0</td>
<td>1.6110</td>
<td>100.0</td>
<td>0.93018</td>
<td>250.0</td>
<td>0.51903</td>
</tr>
<tr>
<td>19.0</td>
<td>1.5393</td>
<td>105.0</td>
<td>0.91701</td>
<td>255.0</td>
<td>0.50584</td>
</tr>
<tr>
<td>20.0</td>
<td>1.4619</td>
<td>110.0</td>
<td>0.90378</td>
<td>260.0</td>
<td>0.49262</td>
</tr>
<tr>
<td>21.0</td>
<td>1.3885</td>
<td>115.0</td>
<td>0.88036</td>
<td>265.0</td>
<td>0.47947</td>
</tr>
<tr>
<td>22.0</td>
<td>1.3147</td>
<td>120.0</td>
<td>0.86705</td>
<td>270.0</td>
<td>0.46623</td>
</tr>
<tr>
<td>23.0</td>
<td>1.2602</td>
<td>125.0</td>
<td>0.85371</td>
<td>275.0</td>
<td>0.45280</td>
</tr>
<tr>
<td>24.0</td>
<td>1.2136</td>
<td>130.0</td>
<td>0.85019</td>
<td>280.0</td>
<td>0.43929</td>
</tr>
</tbody>
</table>

285.0       0.42568  
290.0       0.41189  
295.0       0.39796  
300.0       0.38393
2.1 Controls, Indicators and Connectors

This section of the manual contains functional descriptions of the 110 and 120 front and rear panels. It also includes specific operational information for both Current Sources.

Table 2.1. Functional Description of 110 and 120 Front Panels (refer to Figures 2.1a and 2.1b)

<table>
<thead>
<tr>
<th>Key</th>
<th>Name</th>
<th>Functional Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>POWER</td>
<td>Toggle switch for turning ac power on/off.</td>
</tr>
<tr>
<td>2</td>
<td>ON</td>
<td>LED indicator is lit when ac power is on.</td>
</tr>
<tr>
<td>3</td>
<td>OUTPUT</td>
<td>LED is on continuously when the Source is supplying an output current of either polarity.</td>
</tr>
<tr>
<td>4</td>
<td>OFF</td>
<td>LED flashes when the 110/120 reaches its 12V (minimum) compliance level.</td>
</tr>
<tr>
<td>5</td>
<td>NORM</td>
<td>Three position toggle switch to turn the output current off or on. The current can be turned on with normal (NORM) or reverse (REVRS) polarity.</td>
</tr>
<tr>
<td>6</td>
<td>REVS</td>
<td>10-turn potentiometer for trimming the output current. Range of adjustment is approximately 15.0Ω.</td>
</tr>
<tr>
<td>7</td>
<td>SELECTOR</td>
<td>12-position selector switch to determine the output current level. 11 positions for fixed currents and one position for programmable (PRGM) current source.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key</th>
<th>Name</th>
<th>Functional Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>OUTPUT</td>
<td>2 screw terminals for the unit’s output current. The left terminal is positive (+) for normal (NORM) polarity selection.</td>
</tr>
<tr>
<td>9</td>
<td>CHND</td>
<td>Screw terminal connection for earth/chassis ground.</td>
</tr>
<tr>
<td>10</td>
<td>PROGRAM</td>
<td>2 screw terminals for connecting the unit’s programming resistor. Refer to Section 2.2 for details on using a programming resistor.</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>Unit’s serial number.</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>Unit’s ac power fuse.</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Unit’s ac power cord.</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>Unit’s ac power and fuse data.</td>
</tr>
</tbody>
</table>
SUPERCONDUCTIVITY

consideration shows that to produce the maximum 'persistent'
current in a ring in zero field, it is necessary to cool the ring only in
the field corresponding to the point \( A \) in fig. 12 and then to remove
this field; if the ring had been cooled in zero field, it would be
necessary to put on and remove a field at least as large as that
corresponding to \( Y \).

For completeness it is useful to record here the equations for
the boundary curve obtained with the better approximation of
Schubnikow and Chotkevitch (1936). These are

\[
i = \mp \alpha [H_0 - H_0(1 \pm \delta)](1 \pm \beta),
\]

where \( \alpha = a/4b \) and \( \beta = 2a(1 + \ln 2/\alpha) \); the lower signs are to be
taken when the field is critical at the outer rim, and the upper signs
when the field is critical at the inner rim \( (H_0 \) must be taken negative
for the sections in the left-hand part of the diagram). The observed
magnetic moment will of course also contain a term due to the
magnetic properties of the ring with no total current (p. 27). When
\( \alpha \) is no longer small compared with unity no useful explicit formulae
can be given, but numerical computations for this general torus
have been made by de Launay (1949) and verified experimentally
by Dolecek and de Launay (1949).

2.7. Miscellaneous non-ellipsoidal shapes

In general, no mathematical treatment can be given of the
magnetic behaviour of specimens whose shape is not at least
approximately ellipsoidal, but certain features of the behaviour of
such specimens are of practical interest, and we shall give a brief
account of the various experimental investigations which have been
made.

The behaviour of short cylinders of various length to diameter
ratios has been studied by Shoenberg (1937a), and some typical
magnetization curves for the field parallel and perpendicular to the
cylinder axis are shown in fig. 13. For both settings, the curve no
longer turns over quite so sharply as for an ellipsoid; this can be
interpreted as due to an inhomogeneous distribution of \( H \) in the
cylinder. Thus probably there is no longer a macroscopically
uniform intermediate state during the destruction of supercondu-
ctivity, but some more complicated mixture of superconducting
and normal regions, which varies in composition through the
specimens. For the parallel setting, when the field is reduced below
\( H_c \) there is very marked hysteresis rather reminiscent of that found
for a ring (fig. 12). The probable explanation of this hysteresis is
that, owing to the inhomogeneous field distribution, a ring on the
surface of the specimen is able to become superconducting earlier
than the rest of the specimen as the field is being reduced. For
further reduction of the external field a current is induced in this
ring, thus keeping the flux constant in the interior regions and
preventing them from becoming superconducting again.

![Fig. 13. Magnetization curves of a short tin cylinder (Shoenberg, 1937a).]  
(a) Field parallel to cylinder axis. (b) Field perpendicular to cylinder axis. The discontinuous reappearance of magnetization
(indicated by the broken lines) is discussed on p. 121.

Though the formation of such rings has not yet been deduced
from theoretical knowledge of the field distribution, but has merely
been postulated \( ad \ hoc \) to explain the hysteresis, there is some other
experimental evidence which supports this qualitative explanation.
It was found that the hysteresis of fig. 13 a could be considerably
reduced by rounding off the rims of the cylinder, which probably
made the field distribution more homogeneous, and in terms of our
explanation thus delayed the formation of a superconducting ring.
Moreover, there was practically no hysteresis at all if the field was
perpendicular, instead of parallel, to the axis of the cylinder
(fig. 13 b); in this case there is no symmetry round the field direction.
and so it is impossible for the specimen to become superconducting all round a section normal to the field simultaneously.

The curves of fig. 13, and in particular the form of the hysteresis, were found to be practically independent of temperature, i.e. the magnetization curves at different temperatures could be brought into coincidence by a mere change of scale. This temperature independence provides a practical means of distinguishing the hysteresis due to shape from that due to impurities (see § 2.8), which is usually temperature-dependent.

Somewhat similar hysteresis effects have been found in flat plates transverse to an applied field by Aleksyevsky (1941a, 1946), who has studied in detail the topography of the frozen-in flux, and by Andrew and Lock (1950) (other aspects of whose experiments will be discussed in § 4.5). Here, presumably, the hysteresis was again due to the difficulty of suitably rounding off the plate edges, and it was found, indeed, that electrolytic etching which smoothed the edges did reduce the hysteresis. Another specimen shape that has been studied in some detail is the hollow sphere (Mendelssohn and Babbitt, 1935; Shalnikov, 1940a, 1942), for which again complicated hysteresis effects are found.

It is possible that somewhat different hysteresis mechanism may occasionally be relevant. In some measurement on transverse cylinders by Désirant and Shoenberg (1948a), the hysteresis was usually very slight, but it was noticed that the frozen-in moment could be made much larger if the field was switched off suddenly from a high value instead of being reduced gradually (as was the usual procedure). This suggests that if the field is reduced too rapidly there is not sufficient time for the flux to get out of some parts of the specimen, and it is trapped in normal regions by closed superconducting circuits surrounding them. This interpretation is supported by the fact that time-lag effects have been observed in the intermediate state region. Thus de Haas, Engelskes and Guinier (1937) found that after a sudden change of applied field, the field distribution in a sphere assumes its final form only gradually (the time lag was of order 15 min. at $H_e$ and negligible near $H_e$). Mendelssohn and Pontius (1946a, b) have found that such

* The time effects occurring in the transition of a long cylinder to the superconducting state after "supercooling" are discussed in § 4.6.3.

2.8. Magnetic properties of alloys

Apart from the pure element superconductors whose properties we have been discussing up to now, many alloys are known to become superconducting at low temperatures (see Appendix I, Tables X and XI). Roughly speaking it is possible to divide these alloys into two classes:

1) Alloys of the superconducting elements either with each other, or with elements which are not at present known to become superconducting and which can become superconducting in a large range of concentrations; in these the superconducting property cannot be said to be characteristic of any particular composition corresponding, for instance, to a chemical compound. The transition temperatures are of much the same order of magnitude as for simple superconductors, but the transition is spread over a relatively large temperature range (of order $10^4$ K.). In the literature it is usual to give the transition temperature as that for which the resistance has half its full value, but it should be remembered that this temperature is in general sensitive to the strength of the measuring current. There is no universal rule as to how the transition temperature depends on the composition, but often (particularly for addition of bismuth) the transition temperature...
extra area lies, it is evident that the theory (based on reversible processes) cannot be expected to fit this part of the curve. The strong frozen-in moments found in all these disks are probably associated with the edge effects discussed in § 2.7, and could in fact be appreciably reduced by electrolytic polishing, which smooths the edges. The nature of these frozen-in moments has been studied also by Alekseyevski (1946). Andrew and Lock (1950) also made experiments on the resistance of a thin plate (rectangular instead of circular, however), and the results are shown in fig. 47, together with the magnetization curve of a similar specimen. There is no longer the close correlation between resistance and magnetization results that was found for a wire, the resistance returning at a much higher field (strongly dependent on current) than that at which the magnetization starts to fall. This must mean that the normal regions in the early part of the intermediate state do not reach right across the path of the current flow, and shows that resistance in the intermediate state is by no means inevitable.

4.6. Supercooling

4.6.1. Hysteresis due to supercooling (see also § 5.2.1). We have up to now spoken as if there were no hysteresis in an 'ideal' magnetization curve (such as fig. 6a). Actually experiments on very pure specimens do show a hysteresis which becomes more marked the more 'ideal' is the specimen. This hysteresis (which is indicated by the broken lines in figs. 13 and 44) is in many ways reminiscent of the supercooling of a vapour below its liquefaction temperature, and we shall refer to it as 'supercooling', even though usually it is the magnetic field rather than the temperature which is varied. When the field is reduced from above $H_n$ (or the temperature lowered in a steady field) the specimen stays in the normal state, until at a field appreciably less than $H_n$, a sudden expulsion of flux takes place and the specimen goes into either the intermediate state or, if it has a small enough demagnetization coefficient, directly into the pure superconducting state.

Fig. 47. (a) Magnetization curve, (b) resistance-field curve, for a rectangular tin plate $7.8 \times 10^{-3} \text{cm.}$ thick and $0.1 \times 0.73 \text{cm.}$ for (a) and $0.1 \times 3 \text{cm.}$ for (b) at $3.51^\circ \text{K.}$ (Andrew and Lock, 1950).

Fig. 48. Magnetization curve of an aluminium sphere at $1.16^\circ \text{K.}$ illustrating large supercooling effect (Shoenberg, 1946b). 00 increasing fields; 4 decreasing fields. The scale of magnetization was not calibrated.

In the early measurements with tin specimens only slight supercooling was observed (of order 2 or 3 % of $H_n$) (Mendelssohn and Pontius, 1936a; Shoenberg, 1937a), but a much more marked effect has been found with pure aluminium (Shoenberg, 1946b), as illustrated in fig. 48, and quite recently Faber has been able to demonstrate marked effects in tin by using special techniques (see § 4.6.3). Similar effects had been observed also in the early Leiden measurements on resistance (Sizzo, de Haas and Onnes, 1925; de Haas, Sizzo and Onnes, 1925; de Haas and Voogd, 1928, 1931 a).

* The temperature was originally given as $1.10^\circ \text{K.}$, but a source of error has since been pointed out by Goodman and Mendoza (1951) and an adjustment to the 1949 scale has also been made.
but were influenced strongly by the method of attaching the potential leads and so were thought for some time not to be fundamental (see de Haas, 1933).

4.6.2. Theoretical discussion. H. London (1935) first suggested that such effects might be expected if there were a surface-free energy of suitable magnitude at the boundary between superconducting and normal phases, and that just as in the analogous problem of the vapour-liquid transition, supercooling occurs because of the difficulty of growth of a nucleus of the stable phase.

We shall first contrast briefly the consequences of a positive and a negative interphase surface energy. In fact, we have already seen that a positive surface energy has to be assumed to explain the features of the intermediate state in a pure metal, but as we shall see below there may be small local regions where an interface would have negative surface energy even in a pure metal. If the interphase surface energy is negative, the state of lowest free energy is one in which there is as much splitting into two phases as possible; thus in a field lower than critical the metal would consist of a superconducting matrix broken up by many very thin normal laminae, while above the critical field it would consist of a normal matrix broken up by many very thin superconducting laminae. Naively speaking 'many' should mean 'infinitely many' and 'thin' should mean 'infinitely thin', but actually, of course, the concept of surface energy loses its macroscopic meaning if the laminae become too crowded or too thin, and it is impossible to predict the detailed nature of the splitting without a detailed understanding of superconductivity. For our purposes the qualitative statement that splitting up would occur is sufficient; this splitting would presumably lead to abnormally low resistance persisting above the critical field, and it was just because this did not seem characteristic of 'ideal' conditions that H. London (1935) postulated the existence of a positive rather than a negative interphase surface energy.*

If the surface energy is positive, no superconducting phase should

* If the slight penetration of a magnetic field into the superconducting phase (see Chapter 5) is above considered it can be easily seen that this leads to a negative interphase surface energy $\lambda H_0^2 \sigma$, where $\lambda$ is the penetration depth. As will be explained in § 6.5.3 this is however usually overcompensated by a positive energy associated with the gradualness of the configurational change between the two phases.

survive above the critical field, and no normal phase below (provided, of course, we are considering a body of zero demagnetizing coefficient so that the complications of the intermediate state do not intervene). The transition from normal conductivity to superconductivity must, however, start from some small superconducting nucleus in the normal matrix, and in order to understand why supercooling occurs we shall now consider the equilibrium of such a nucleus. Let its volume be $V$ and its surface area $A$, and assume for simplicity that its demagnetizing coefficient is negligible; we shall suppose that the interphase surface free energy, $\alpha$ per unit area, may vary from place to place, but again for simplicity we shall assume that the values of $\alpha$ and its variation $\partial \alpha / \partial n$ along the normal to the surface of the nucleus are constant over the surface of the nucleus.* If the nucleus is to be in equilibrium at a particular size, the free energy of the specimen must remain unchanged for a small growth of the nucleus; if the free energy increases, the nucleus will shrink, while, if it decreases, the nucleus will grow. Thus, assuming that $\alpha$ is independent of the field, the conditions for growth, equilibrium or shrinkage of a nucleus are

\[
\left( g_n - g_s - \frac{H_0^2}{8\pi} \right) \frac{\partial V}{\partial n} = \alpha \delta A + \frac{\partial \alpha}{\partial n} \delta V,
\]

where $g_n$ and $g_s$ are the free energies per unit volume of the two phases (in zero field for the superconducting phase), and the upper inequality refers to growth. Substituting $H_0^2/8\pi$ for $(g_n - g_s)$ from (3.1) and using the notation $\Delta = 6m/4\pi\alpha^2$ as already defined in (4.1), this becomes

\[
\left( 1 - \frac{H_0^2}{\Delta} \right) \frac{\Delta}{\pi} \delta A = \frac{\partial \Delta}{\partial A} \delta V,
\]

(4.10)

where $d$ is the appropriate small dimension of the superconducting nucleus defined by $d = 4V/\partial A$; it should be noted that the precise meaning of $d$ depends on the manner of growth considered—for a cylinder growing radially, $d$ is just the radius, but for a cylinder

* It should be emphasized that these assumptions are essentially oversimplifications, since the shape of the nucleus should also be considered as a variable; thus for positive $\alpha$ the surface energy is least for a spherical shape, but the demagnetization energy is least for a long shape or a lamina. Moreover the concept of a well-defined surface boundary implied in the discussion is unlikely to be quite valid in a discussion of small regions in a non-uniform material.