Today:
• **Today, Chapter 11:**
  • Angular velocity and Angular acceleration vectors
  • Torque and the Vector Cross Product
  • Angular Momentum
  • Conservation of Angular Momentum
  • Gyroscopes and Precession

Funny animation of skater from [http://i.imgur.com/VgBOkoc.gif](http://i.imgur.com/VgBOkoc.gif)

**Torque of a quick push**

A student gives a quick push to a puck that can rotate in a horizontal circle on a frictionless table. *After* the push has ended, the puck’s angular speed

A. Steadily increases.
B. Increases for awhile, then holds steady.
C. Holds steady.
D. Decreases for awhile, then holds steady.
E. Steadily decreases.
Class 18 **Optional** Preclass Quiz on MasteringPhysics

- This was due this morning at 10:00am
- 94% of students got: A uniform disk, a uniform hoop, and a uniform solid sphere are released at the same time at the top of an inclined ramp. They all roll without slipping. **They reach the bottom of the ramp in the order:** sphere \( \left( \frac{2}{5} MR^2 \right) \), disk \( \left( \frac{1}{2} MR^2 \right) \), hoop \( MR^2 \).

- Basically the larger the rotational inertia, the slower the object rolls down the hill (because it requires more static friction to get it rolling).

Class 18 **Optional** Preclass Quiz Student Comments

- *Would you rather be rick rolled everyday for the rest of your life or curve up our midterm marks? Choose only one.*
- **Harlow answer:** I choose the curve for sure.
- *My grades are telling me to be a trophy husband, but my looks are telling me to study harder.*
- *I found the right hand rule confusing and would like that to be explained. The section about gyroscopes was interesting.*
- *If two people are rotating but in opposite directions, is one of their angular momentum the negative of the other?*
- **Harlow answer:** Correct.
Class 18 **Optional** Preclass Quiz Student Comments

- **[part 1:]** Since static friction is always associated with not moving is it impossible for static friction to do work?
- **Harlow answer:** Correct. Static friction from a non-moving surface cannot do work.
- **[part 2:]** If the net force causing a car to move forwards is static friction what force is doing the work on the car?
- **Harlow answer:** The car is doing work on itself! Chemical energy is being converted to mechanical energy internally. The static friction is important only to provide a rolling without slipping constraint. Another way to look at this is that energy is not being transferred from the road to the car.

---

**The Vector Description of Rotational Motion**

- One-dimensional motion uses a scalar velocity $v$ and force $F$.
- A more general understanding of motion requires vectors $\vec{v}$ and $\vec{F}$.
- Similarly, a more general description of rotational motion requires us to replace the scalars $\omega$ and $\tau$ with the vector quantities $\vec{\omega}$ and $\vec{\tau}$.
- Doing so will lead us to the concept of *angular momentum*. 
The Angular Velocity Vector

- The magnitude of the angular velocity \( \mathbf{\omega} \) vector is \( \omega \).
- The angular velocity vector points along the axis of rotation in the direction given by the right-hand rule as illustrated.

The Cross Product of Two Vectors

The scalar product (dot) is one way to multiply two vectors, giving a scalar. A different way to multiply two vectors, giving a vector, is called the cross product.

If vectors \( \mathbf{A} \) and \( \mathbf{B} \) have angle \( \alpha \) between them, their cross product is the vector:

\[
\mathbf{A} \times \mathbf{B} \equiv (AB \sin \alpha, \text{in the direction given by the right-hand rule})
\]

The cross product is perpendicular to the plane.
The Right-Hand Rule

The cross product is perpendicular to the plane of $\vec{A}$ and $\vec{B}$. The right-hand rule for the direction comes in several forms. Try them all to see which works best for you.

Note that $\vec{B} \times \vec{A} \neq \vec{A} \times \vec{B}$. Instead, $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$.

The Torque Vector

We earlier defined torque $\tau = rF\sin\phi$. $r$ and $F$ are the magnitudes of vectors, so this is a really a cross product:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

A tire wrench exerts a torque on the lug nuts.
Angular Momentum of a Particle

A particle of mass $m$ is moving. The particle’s momentum vector makes an angle $\beta$ with the position vector.

\[
\mathbf{L} \equiv \mathbf{r} \times \mathbf{p} = (mrv \sin \beta, \text{ direction of right-hand rule})
\]

Angular Momentum of a Particle

Why this definition? \[\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}\]

If you take the time derivative of $\mathbf{L}$ and use the definition of the torque vector, you find:

\[
\frac{d\mathbf{L}}{dt} = \mathbf{r}_{\text{net}}
\]

Torque causes a particle’s angular momentum to change. This is the rotational equivalent of \[d\mathbf{p}/dt = \mathbf{F}_{\text{net}}\] and is a general statement of Newton’s second law for rotation.
Angular Momentum of a Rigid Body

For a rigid body, we can add the angular momenta of all the particles forming the object. If the object rotates

- on a fixed axle, or
- about an axis of symmetry

then it can be shown that

\[ \vec{L} = I \vec{\omega} \quad \text{(rotation about a fixed axle or axis of symmetry)} \]

And it’s still the case that \( \frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}} \).

Angular Momentum

- Angular momentum
  \[ \vec{L} = I \vec{\omega} \]
  - This is analogous to
  Linear momentum = mass \( \times \) velocity
  \[ \vec{p} = m\vec{v} \]
• A bicycle is traveling toward the right.
• What is the direction of the angular momentum of the wheels?
  A. left
  B. right
  C. into page
  D. out of page
  E. up

Conservation of Angular Momentum

An isolated system that experiences no net torque has
\[
\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}} = \vec{0}
\]
and thus the angular momentum vector \( \vec{L} \) is a constant.

\[
\vec{L}_i = \vec{L}_f
\]

Rotating body: \( I_i \omega_i = I_f \omega_f \)
The Law of Conservation of Momentum

- If there is no net external force on a system, then its momentum is a constant.

The Law of Conservation of Energy

- If there is no work or heat being exchanged with a system and its surroundings, then its energy is constant.

The Law of Conservation of Angular Momentum

- If there is no net external torque on a system, then its angular momentum is a constant.

Angular Momentum

CHECK YOUR NEIGHBOR

Suppose you are swirling a can around and suddenly decide to pull the rope in halfway; by what factor would the speed of the can change?

A. Double
B. Four times
C. Half
D. One-quarter
Suppose you are swirling a can around and suddenly decide to pull the rope in halfway; by what factor would the speed of the can change?

A. Double
B. Four times
C. Half
D. One-quarter

Conservation of Angular Momentum

Example:

- When the professor pulls the weights inward, his rotational speed increases!
Linear / Rotational Analogy

<table>
<thead>
<tr>
<th>Linear</th>
<th>Rotational Analogy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{s}$, $\vec{v}$, $\vec{a}$</td>
<td>$\theta$, $\omega$, $\alpha$</td>
</tr>
<tr>
<td>Force: $\vec{F}$</td>
<td>Torque: $\tau$</td>
</tr>
<tr>
<td>Mass: $m$</td>
<td>Rotational Inertia: $I$</td>
</tr>
</tbody>
</table>

- Newton’s 2nd law: $\vec{a} = \frac{\vec{F}_{net}}{m}$
- Kinetic energy: $K_{cm} = \frac{1}{2}mv^2$
- Momentum: $\vec{p} = m\vec{v}$
- $\alpha = \frac{\tau_{net}}{I}$
- Rotational energy: $K_{rot} = \frac{1}{2}I\omega^2$
- Momentum: $\vec{L} = I\vec{\omega}$

Two buckets spin around in a horizontal circle on frictionless bearings. Suddenly, it starts to rain. As a result,

A. The buckets speed up because the potential energy of the rain is transformed into kinetic energy.
B. The buckets continue to rotate at constant angular velocity because the rain is falling vertically while the buckets move in a horizontal plane.
C. The buckets slow down because the angular momentum of the bucket + rain system is conserved.
D. The buckets continue to rotate at constant angular velocity because the total mechanical energy of the bucket + rain system is conserved.
E. None of the above.
Consider a horizontal gyroscope, with the disk spinning in a vertical plane, that is supported at only one end of its axle, as shown. You would expect it to simply fall over—but it doesn’t.

Instead, the axle remains horizontal, parallel to the ground, while the entire gyroscope slowly rotates in a horizontal plane.

This steady change in the orientation of the rotation axis is called precession, and we say that the gyroscope precesses about its point of support.

The precession frequency $\Omega$ is much less than the disk’s rotation frequency $\omega$. 

\[
\frac{m_1 \omega_f}{2} = \frac{m_1}{2} \omega_f + m_2 \omega_f
\]

\[
\frac{9}{2} \omega_f = \frac{9}{2} \omega_f + 1 \omega_f = (1 + 1) \omega_f
\]

\[
\omega_f = \frac{\omega_f}{2} = 100 \text{ rpm}
\]
Gravity on a Nonspinning Gyroscope

- Shown is a nonspinning gyroscope.
- When it is released, the net torque is entirely gravitational torque.
- Initially, the angular momentum is zero.
- Gravity acts to increase the angular momentum gradually in the direction of the torque, which is the $\hat{z}$-direction.
- This causes the gyroscope to rotate around $x$ and fall.

Gravity on a Spinning Gyroscope

- Shown is a gyroscope initially spinning around the $z$-axis.
- Initially, gravity acts to increase the angular momentum slightly in the direction of the torque, which is the $\hat{i}$-direction.
- This causes the gyroscopes angular momentum to shift slightly in the horizontal plane.
- The gravitational torque vector is always perpendicular to the axle, so $d\vec{L}$ is always perpendicular to $\vec{L}$. 

The torque is perpendicular to $\vec{r}$.

The initial angular momentum is zero.

The gyroscope falls.

The angular momentum increases in the direction of the torque.

The torque hasn’t changed.

The gyroscopes precesses.

The angular momentum changes direction but not magnitude.

View from above.

$\vec{L}$ is always perpendicular to $\vec{L}$. 

Precession of a Gyroscope

- The precession frequency of a gyroscope, in \( \text{rad/s} \), is
  \[ \Omega = \frac{Mgd}{I\omega} \]
- Here \( M \) is the mass of the gyroscope, \( I \) is its rotational inertia, and \( d \) is the horizontal distance of the center of mass from the support point.
- The angular velocity of the spinning gyroscope is assumed to be much larger than the precession frequency; \( \omega \gg \Omega \).

Nuclear Magnetic Resonance

- A proton in the nucleus of an atom is like a little spinning top.
- When placed in a strong static magnetic field, the magnetic force produces a torque on the proton, which causes it to precess.
- The precession frequency is in the radio-frequency range, which allows the proton to absorb and re-emit radio-waves.
- This allows doctors to image inside the human body using completely harmless radio waves.
Before Class 19 on Monday

- Please read chapter 12 on Static Equilibrium. The preclass quiz is due Monday morning.
- Problem Set 8 on Chapters 10 and 11 is due Tuesday Nov. 24 at 11:59pm. Problem sets have Tuesday deadlines now!
- Something to think about over the weekend: The supports to the diving board provide a vertical force on the board so the girl will not fall. What are the directions of the force on the board at point 1 and point 2: up or down? Why?